Journal of
Intelligent Material
Systems and Structures

Article

Journal of Intelligent Material Systems and Structures
24(6) 729–744

DOI: 10.1177/1045389X12448447
jim.sagepub.com
SAGE

Eigensolution of piezoelectric energy harvesters with geometric discontinuities: Analytical modeling and validation

Adam M Wickenheiser

Abstract
Although cantilevered beams are the most prolific design for resonant piezoelectric energy harvesters, other topologies have been studied for their compactness or conformability to their host structures' geometry. These more complex structures have been analyzed using custom analytical models developed from the first principles or finite-element methods to compute their eigensolutions and piezoelectric coupling effects. This article discusses the use of the transfer matrix method to derive analytical solutions to beam structures with pointwise discontinuities, bends, or lumped inertias between members or at the tip. Euler–Bernoulli beam theory is used to derive transfer matrices for the uniform beam segments, and point transfer matrices are derived to handle discontinuities in the structure between beam segments. The eigensolution of the transfer matrix is shown to produce the natural frequencies and mode shapes for these structures. Subsequently, the electromechanical coupling effects are incorporated, and the base excitation problem is considered. Parametric case studies are provided for beam structures with varying piezoelectric layer coverage and angle between members. Finally, these results are compared to finite-element solutions using COMSOL, and the modeling discrepancies are discussed. Based on the favorable comparison between these two methods, the utility and accuracy of the transfer matrix method are proven.

Keywords
Energy harvesting, piezoelectric, sensor

Introduction

A major problem with resonant vibration–based energy harvesters is that their peak power occurs near the mechanical resonant frequencies of the transducer. In order to shrink the size and mass of these devices while reducing their natural frequencies, a variety of techniques have been employed. For example, changing the standard cantilevered beam geometry and manipulating the mass distribution along the beam have been investigated. Designing the structure with a continuously varying cross section (Dietl and Garcia, 2010; Reissman et al., 2007; Roundy et al., 2005) and tuning the ratio of the tip mass to beam mass (Dietl and Garcia, 2010; Wickenheiser, 2011a) have been shown to increase the electromechanical coupling of the device, a measure of how much energy is successfully harvested per cycle. Changing the number and location of piezoelectric patches along the beam can also increase the coupling by focusing on where the greatest strain energy is found (Guyomar et al., 2005; Wu et al., 2009). Multibeam structures can shrink the overall dimensions of a structure by folding it in on itself while minimizing the increase in fundamental natural frequency (Karami and Inman, 2011); they can also increase effective bandwidth by clustering natural frequencies together (Berdy et al., 2011; Erturk et al., 2009). A nonlinear technique called "frequency up-conversion" also shows promise to boost power at frequencies well below resonance (Murray and Rastegar, 2009; Tieck et al., 2006; Wickenheiser and Garcia, 2010b). Despite the prevalence of widely varying designs, no single analytic

Department of Mechanical & Aerospace Engineering, George Washington University, Washington, DC, USA

Corresponding author:
Adam M Wickenheiser, Department of Mechanical & Aerospace Engineering, George Washington University, 801 22nd St. NW, Washington, DC 20052, USA.
Email: amwick@gwu.edu